Effects of cross-phase modulation on phase jitter in soliton systems with constant dispersion

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In wavelength-division-multiplexed communication systems phase jitter is driven by amplifier noise and mediated by cross-phase modulation. The variational method is used to derive formulas for the absolute phase variance of an ensemble of solitons and the relative phase variance of ensembles of neighboring solitons. The predictions of these formulas are consistent with the results of numerical simulations. © 2003 Optical Society of America

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In differential-phase-shift-keyed communication systems the error-free transmission distance is limited by phase jitter, in which power shifts caused by amplifier noise are converted into phase shifts by nonlinear phase modulation.[1] Phase jitter in single-channel systems is mediated by self-phase modulation (SPM). For soliton systems the soliton-perturbation[2,3] and variational[4,5] methods have been used to study this phenomenon.[6–8] Phase jitter in multichannel or wavelength-division-multiplexed systems is mediated by SPM and by cross-phase modulation (CPM). In this Letter we use the variational method to study the effects of CPM on phase jitter in soliton systems with constant dispersion.

Wave propagation in a fiber is governed by the nonlinear Schrödinger equation

$$\partial_z A = (g - \alpha)A/2 - i\beta\partial_{tt}A/2 + i\gamma|A|^2 A + R\,, \qquad (1)$$

where A is the wave amplitude, g is the amplification (gain) rate, and α, β, and γ are the fiber loss, dispersion, and nonlinearity coefficients, respectively. The effects of amplifier noise are modeled by the source term R, which is a random function of distance and time. For uniformly distributed amplification, $g = \alpha$.

The variational method is based on the assumption that each pulse propagates with a self-similar shape. If one applies a multiple-pulse shape ansatz to the Lagrangian density associated with Eq. (1) and time averages the Euler–Lagrange equations, one converts a partial differential equation for the wave amplitude into a system of ordinary differential equations for the shape parameters of the pulses. For the ansatz

$$A_i(z,t) = a_i \exp[i\phi_i - i\omega_i(t - t_i) - (1 - ic_i)(t - t_i)^2/2\tau_i^2], \qquad (2)$$

where a_i, ϕ_i, ω_i, t_i, c_i, and τ_i are the amplitude, phase, frequency, time delay, chirp, and width, respectively, of a pulse in channel i, the energy $E_i = \pi^{1/2}a_i^2\tau_i$. The variational equations associated with ansatz (2) are stated in Refs. 8 and 9.

In the absence of noise an isolated pulse propagates unchanged, provided that the soliton condition

$$\gamma E_i/(2\pi)^{1/2}\tau_i = |\beta|/\tau_i^2 \qquad (3)$$

is satisfied. In such an equilibrium, the cumulative SPM is

$$\phi_i(z) = 3\gamma E_i z/4(2\pi)^{1/2}\tau_i\,. \qquad (4)$$

Consider a 10-Gbit/s system in which $\alpha = 0.21$ dB/km, $\beta = -0.30$ ps^2/km [$D = 0.38$ ps/(km-nm)], which is comparable to the spatially averaged dispersion of current dispersion-managed systems, and $\gamma = 1.7$/(km-W). Then an exact (sech) soliton (upon which the nonlinear Schrödinger simulations are based) with a full width at half-maximum of 30 ps has energy $E_i = 21$ fJ. For a Gauss soliton of the same energy, $\tau_i = 21$ ps. If the system length is 3.0 Mm, the cumulative phase shift is 1.5 rad.

In the presence of noise the shape parameters of an isolated pulse undergo a noise-driven random walk. Of prime concern is the phase, which evolves according to the equation

$$d_z\phi_i = \delta\phi_z + \beta/2\tau_i^2 + 5\gamma E_i/4(2\pi)^{1/2}\tau_i\,. \qquad (5)$$

By solving Eq. (5) and the associated equations for the energy and width, one can show that the phase variance

$$\langle\delta\phi_i^2(z)\rangle = (S_z/E_i)\{c_1 z + c_3[3\gamma E_i/4(2\pi)^{1/2}\tau_i]^2 z^3\}\,, \qquad (6)$$

where $\langle\rangle$ denotes an ensemble average, $c_1 = 0.75$, and $c_3 = 1.9$ or $c_3 = 2.7$, depending on whether the distance is shorter or longer than the dispersion distance ($\tau_i^2/|\beta| = 1.5$ Mm), respectively.[8] The source strength $S_z = n_{\rm sp}h\nu g$, where $n_{\rm sp}$ is the spontaneous noise factor (1.1 for Raman amplification) and $h\nu$ is the photon energy.

In a wavelength-division multiplexing system the phase shift caused by a pulse in channel j grows according to the equation

$$\frac{d\phi_i}{dz} = \frac{\gamma E_j}{[\pi(\tau_i^2 + \tau_j^2)]^{1/2}}\left[\frac{3\tau_i^2 + 2\tau_j^2}{\tau_i^2 + \tau_j^2} - \frac{2\tau_i^2(t_i - t_j)^2}{(\tau_i^2 + \tau_j^2)^2}\right]\exp\left[-\frac{(t_i - t_j)^2}{\tau_i^2 + \tau_j^2}\right]. \qquad (7)$$

Provided that the pulse parameters do not change significantly during the collision and that $d_z(t_i - t_j) \approx \beta\omega_{ij}$, where $\omega_{ij} = \omega_i - \omega_j$ is the equilibrium

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frequency difference, the cumulative phase shift

$$\delta\phi_i \approx 2\gamma E_j/|\beta\omega_{ij}| \tag{8}$$

for arbitrary values of E_i, τ_i, and τ_j. Expression (8) is consistent with the exact (inverse-scattering) result $\delta\phi_i = 2\tan^{-1}(\gamma E_j/|\beta\omega_{ij}|)$.[10] For a channel spacing of 100 GHz the phase shift associated with a nearest-neighbor collision is 0.37. There are $n_{ij} = z/z_{ij}$ such collisions, where $z_{ij} = T/|\beta\omega_{ij}|$ is the distance between collisions and $T = 100$ ps is the bit period. CPM does not depend on which pulse overtakes the other. Provided that they have the same energy, colliding pulses in channel $i - j$ have the same effect on a reference pulse in channel i as colliding pulses in channel $i + j$. For simplicity of notation, these colliding channels are referred to as channels $-j$ and j, respectively.

Consider an ideal (noiseless) system in which the energies are all equal to the mean energy E. For such a system the cumulative phase shift associated with each pair of channels (j and $-j$)

$$\delta\phi_i(z) = 4\gamma Ez/T\,. \tag{9}$$

For the aforementioned parameters the cumulative phase shift is 4.2. Notice that Eq. (9) does not depend on the channel spacing: As the channel spacing increases, the reduction in collision strength is compensated by the increase in the number of collisions. Consequently, CPM is a much stronger effect than SPM. In an ideal system there is no phase jitter.

Now consider a real (noisy) system, in which the energies are distributed about the mean energy. For a collision that is centered at the distance z_c, the energy variance

$$\langle \delta E_j{}^2(z_c)\rangle = \langle \delta E_j{}^2(0)\rangle + 2S_z E z_c\,. \tag{10}$$

The first term on the right-hand side of Eq. (10) represents the variance associated with an imperfect pulse generator, whereas the second term represents the noise-induced variance. In Fig. 1 the (normalized) noise-induced energy deviation is plotted as a function of distance. If the energy deviation produced by the generator does not exceed 1%, the noise-induced deviation will be dominant for distances longer than 0.15 Mm. The resultant dependence of collision-induced phase jitter on noise-induced energy jitter in the neighboring channels distinguishes phase jitter from collision-induced timing jitter.

It follows from expression (8) that the phase variance associated with each collision

$$\langle \delta\phi_i{}^2\rangle = 4\gamma^2\langle \delta E_j{}^2\rangle/|\beta\omega_{ij}|^2. \tag{11}$$

Because the energy perturbations associated with different collisions are independent, the associated phase variances add linearly. Provided that the reference pulse undergoes many collisions as it propagates through the system, one can replace a discrete collision sum by a continuous distance integral. As stated after expression (8), pulses in channel $-j$ have the same effect on the reference pulse as pulses in channel j. It follows from these observations that, for each pair of channels, the cumulative phase variance is

$$\langle \delta\phi_i{}^2(z)\rangle = 8\gamma^2 S_z E z^2/|\beta\omega_{ij}|T\,. \tag{12}$$

In contrast to the phase variance associated with SPM, which grows as z^3 [Eq. (6)], the phase variance associated with CPM grows as z^2. To account for n_c pairs of neighboring channels one multiplies the right-hand side of Eq. (12) by the factor $\sum_{j=1}^{n_c} 1/j \sim 0.58 + \log n_c$. For $n_c = 2$, 4, and 64 the channel factors are 1.50, 2.08, and 4.74, respectively. As the number of channels tends to infinity, so also does the channel factor (albeit slowly). In Fig. 2 the absolute phase variances are plotted as functions of distance for systems with 3, 5, and 9 channels. The analytical curves were obtained from Eq. (12), whereas the numerical curves were obtained by solving the nonlinear Schrödinger equation many (of the order of 10^4) times with noise added to the colliding channels but not to the reference channel. For the parameters of the figure there is good agreement between the analytical predictions and the numerical results. For longer distances the absolute phase variance grows more quickly than Eq. (12) predicts and oscillates as it grows.

In phase jitter mediated by SPM the phase perturbations of neighboring reference pulses are independent,

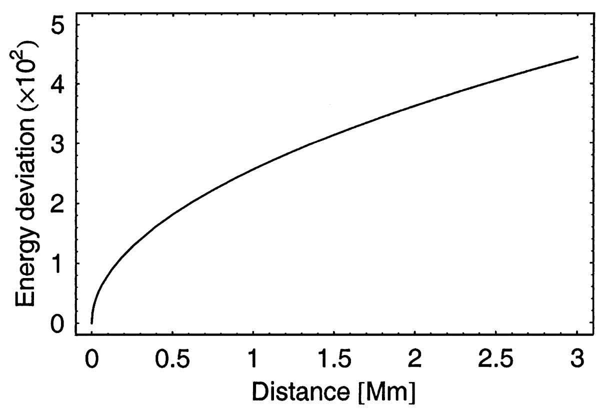

Fig. 1. Standard deviation of the (normalized) noise-induced energy shifts plotted as a function of distance.

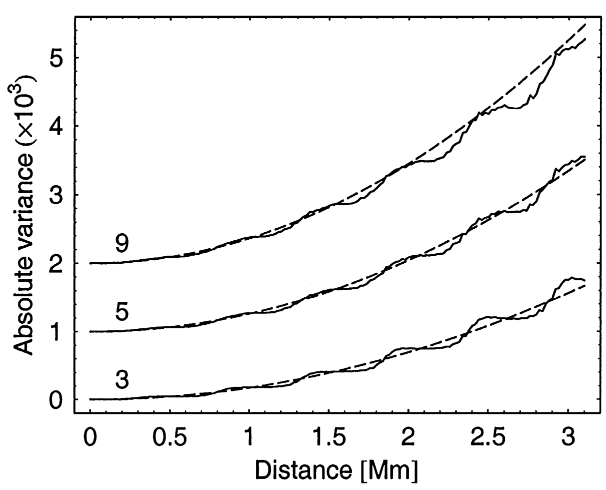

Fig. 2. Absolute phase variances plotted as functions of distance for systems with 3, 5, and 9 channels, separated by multiples of 100 GHz. Dashed curves denote the predictions of Eq. (12); solid curves denote the simulation results. For clarity, the five- and nine-channel curves are displaced upward by 1 and 2, respectively.

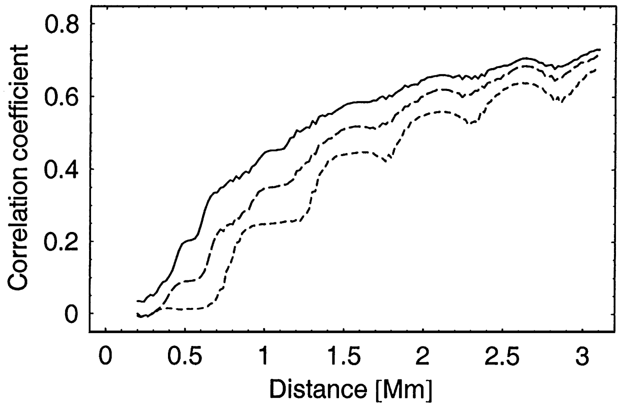

Fig. 3. Phase correlation coefficients plotted as functions of distance for systems with 3, 5, and 9 channels, separated by multiples of 100 GHz. Dotted, dashed, and solid curves denote 3-, 5-, and 9-channel correlations, respectively.

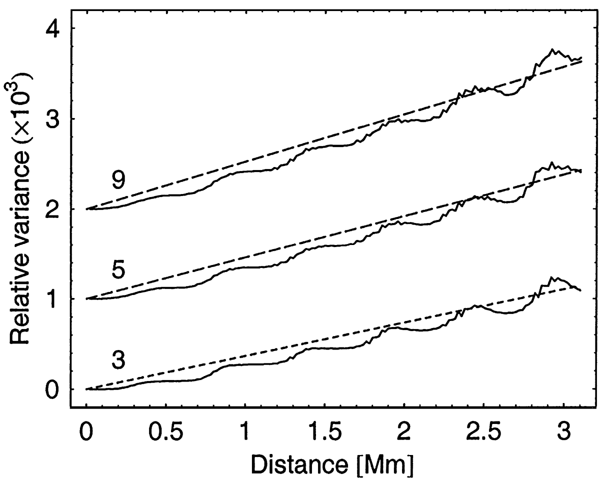

Fig. 4. Relative phase variances plotted as functions of distance for systems with 3, 5, and 9 channels, separated by multiples of 100 GHz. Dashed curves denote the predictions of Eq. (14); solid curves denote the simulation results. For clarity, the 5- and 9-channel curves are displaced upward by 1 and 2, respectively.

so their relative phase variance is larger than their (common) absolute phase variance by a factor of 2. However, in phase jitter mediated by CPM the phase perturbations of neighboring pulses are correlated. The phase correlation coefficients of neighboring pulses obtained from the aforementioned numerical simulations are plotted as functions of distance in Fig. 3. For any number of channels, the correlation between neighboring pulses is significant. As the number of channels increases, the correlation increases and the spatial oscillations in correlation, which occur because the collisions are discrete, not continuous, decrease.

Consider the collision histories of neighboring pulses in channel i. Pulse 1 is overtaken by pulses 1, $2 \ldots n_{ij}$ in channel j. These collisions are centered at $z_{ij}/2, 3z_{ij}/2 \ldots z - z_{ij}/2$, respectively. Pulse 2 collides with pulses 2, $3 \ldots n_{ij} + 1$ in channel j at the aforementioned locations. It follows from expression (8) that

$$\delta\phi_{i1} - \delta\phi_{i2} \propto \sum_{k=1}^{n_{ij}} \delta E_{jk}[(k - 1/2)z_{ij}] - \sum_{k=2}^{n_{ij}+1} \delta E_{jk}[(k - 3/2)z_{ij}]. \quad (13)$$

Only the first collision of pulse 1 and the last collision of pulse 2 are uncorrelated. For $k = 2 \ldots n_{ij}$ each pulse in channel j collides with pulse 2 before it collides with pulse 1. The only difference in the associated values of δE_{jk} is that which accrues in the distance z_{ij}. It follows from these observations that, for each pair of channels, the relative phase variance

$$\langle \delta\phi_i{}^2(z) \rangle = 32\gamma^2 S_z E z/(\beta\omega_{ij})^2. \quad (14)$$

In contrast to the absolute phase variance [Eq. (12)], which grows as z^2, the relative phase variance grows as z. To account for n_c pairs of neighboring channels one multiplies the right-hand side of Eq. (14) by the factor $\sum_{j=1}^{n_c} 1/j^2$. For $n_c = 2$, 4, and 64 the channel factors are 1.25, 1.42, and 1.64, respectively. The channel factor does not exceed 1.65 for any number of channels. In Fig. 4 the relative phase variances are plotted as functions of distance for systems with 3, 5, and 9 channels. The analytical curves were obtained from Eq. (14), whereas the numerical curves were obtained from the aforementioned numerical simulations. There is reasonable agreement between the analytical predictions and the numerical results.

Provided that the phase-shift distribution is Gaussian, the probability of measuring a phase shift of 1.6 is less than 10^{-9} if the standard deviation of the phase shifts is smaller than (the critical deviation) 0.26. At a distance of 3 Mm the (relative) phase deviation mediated by SPM is 0.11, the phase deviation mediated by CPM (9 channels) is 0.041, and the combined phase deviation is 0.12. For the stated parameters, phase jitter does not prevent error-free transmission.

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